Published by Institute of Physics Publishing for SISSA

Received: *May 7, 2008*
Revised: *June 15, 2008*
Accepted: *July 8, 2008*
Published: *July 21, 2008*

Quark mass deformation of holographic massless QCD

Koji Hashimoto,[a] **Takayuki Hirayama,**[bc] **Feng-Li Lin**[c] **and Ho-Ung Yee**[d]

[a] *Theoretical Physics Laboratory, RIKEN, 2-1 Hirosawa, Saitama 351-0198, Japan*
[b] *Physics Division, National Center for Theoretical Sciences, 101, Section 2 Kuang Fu Road, Hsinchu 300, Taiwan*
[c] *Department of Physics, National Taiwan Normal University, 88, Section 4, Ting-Chou Road, Taipei 116, Taiwan*
[d] *The Abdus Salam International Center for Theoretical Physics, Strada Costiera 11, 34014, Trieste, Italy*
E-mail: koji@riken.jp, hirayama@phys.cts.nthu.edu.tw, linfengli@phy.ntnu.edu.tw, hyee@ictp.it

Abstract: We propose several quark mass deformations of the holographic model of massless QCD using the D4/D8/$\overline{\text{D8}}$-brane configuration proposed by Sakai and Sugimoto. The deformations are based on introducing additional D4- or D6-branes away from the QCD D4 branes. The idea is similar to extended technicolor theories, where the chiral symmetry breaking by additional D-branes is mediated to QCD to induce non-zero quark masses. In the D-brane picture as well as the holographic dual gravity description, the quark and the pion masses are generated by novel worldsheet instantons with finite area. We also derive the Gell-Mann-Oakes-Renner relation, and find the value of the chiral condensate in the Sakai-Sugimoto model.

Keywords: Gauge-gravity correspondence, AdS-CFT Correspondence.

Contents

1. Introduction

AdS/CFT correspondence [1] has provided us with a new avenue to compute observables in strongly interacting gauge theories via weakly coupled gravitational descriptions. Even though the correspondence is still a conjecture, many nontrivial computations have been performed for the consistency check of the correspondence. One of the most important applications, and also challenges, would be to analyze real QCD in Nature using the AdS/CFT correspondence.

Recently, there proposed a holographic model of QCD by Sakai and Sugimoto, where the non-Abelian chiral symmetry and its spontaneous breaking is geometrically realized via a D4/D8/$\overline{\text{D8}}$ configuration [2]. The model predictions for the meson spectrum and the couplings between them compare well with experiments with a good accuracy. Baryons in this model were also investigated in [3]. Moreover, this holographic model has an ability of predicting possible glue-ball interactions to mesons as studied in [4]. There are many other QCD observables that have been analyzed in this model, which show that the model is a good approximation to real QCD at least in low energy, large N_c-limit.

Despite its success in many aspects, this model has an apparent shortcoming that the pions are massless. This is because the quarks are massless from the construction. Therefore one of the important questions is to find a deformation of the theory which corresponds to introducing bare quark masses. Once we identify the deformation, we should be able to show that the Gell-Mann-Oakes-Renner (GOR) relation [5] is satisfied, from which the value of the chiral condensate of the Sakai-Sugimoto model can be computed. Pion dynamics would be the most important in holographic QCD because the Sakai-Sugimoto model approaches real QCD in the low energy limit, thus the identification of the quark mass deformation in the model is indispensable.

The chiral quarks appear as the lowest massless modes of the open string stretching between intersecting N_c D4-branes and N_f D8-branes (or $\overline{\text{D8}}$-branes) in the model. Therefore one would naively expect that the quarks become massive if one can realize the situation where the D8 and $\overline{\text{D8}}$-branes do not intersect with the D4-branes. Some of the authors [6] studied such local deformations near the D4-branes (which do not deform the asymptotic configuration away from the D4-branes) to realize the pion mass, but the quarks in that set-up are suggested to be still massless. This is in fact consistent with the proposed relation [7] between the classical value of action in the bulk and the partition function in CFT for the $\text{AdS}_5/\text{CFT}_4$ correspondence, since the value of bulk field at the AdS boundary is related with the strength of coupling, such as the mass, in CFT. There are attempts to include D8-$\overline{\text{D8}}$-brane tachyon in the effective action to explicitly break the chiral symmetry [8]. However, tachyon action is not reliable as there is no consistent truncation of string theory to the tachyon sector. Furthermore, with the tachyon condensation, possible relation to the original Sakai-Sugimoto brane configuration is not obvious.

In this work, we propose new deformations of the Sakai-Sugimoto model which correspond to the introduction of the quark mass. Our deformations have additional D-branes away from the original confining N_c D4-branes, which still have a reasonable field theory interpretation. The original Sakai-Sugimoto model is restored once these additional D-branes are moved to spatial infinity in the bulk. In the field theory view-point, our idea is rather similar to the technicolor model [9], so our construction can be thought of as a holographic realization of technicolor model. In the minimal technicolor model, we have an additional sector to the original QCD, which contains new techni-quarks interacting via new technicolor interactions. The quarks/techni-quarks are massless at UV, but the chiral symmetry (the electroweak symmetry in this case) is spontaneously broken by a techni-quark condensate driven by the strong technicolor gauge interactions. To realize QCD quark mass from this breaking, we need to introduce extended technicolor interactions between quarks and techni-quarks, in addition to the usual weak interactions [10]. This setup is actually close to the holographic dual of extended type technicolor model by applying the D-brane configuration of Sakai-Sugimoto model in [11]. This extended gauge group can often be realized in GUT type construction, where quarks and techni-quarks are in a single multiplet, and the GUT gauge symmetry is broken to QCD and technicolor at some high scale. Through off-diagonal massive gauge bosons, QCD quarks get explicit

JHEP07(2008)089

bare mass term from the techni-quark condensate. From the view-point of QCD, we have a realization of explicit chiral symmetry breaking mass for the quarks.

To realize this idea in our set-up, we introduce additional N' D4-branes (we call them D4$'$-branes) which are parallel to, but separated from the original N_c D4-branes of QCD gauge symmetry. The GUT gauge symmetry is $\mathrm{SU}(N_c+N')$ when the D4$'$-branes are on top of the original D4-branes, and it is broken to $\mathrm{SU}(N_c)\times\mathrm{SU}(N')$ by a Higgs mechanism via separating the D4$'$ from the D4. Massive off-diagonal gauge bosons appear as strings suspended between the D4 and the D4$'$. We add N_f D8/$\overline{\text{D8}}$-branes as in [2] to introduce massless quarks and techni-quarks, which are charged under the chiral symmetry $\mathrm{U}(N_f)\times\mathrm{U}(N_f)$. By assuming a strong technicolor $\mathrm{SU}(N')$ dynamics on the D4$'$-branes, which replaces the D4$'$-branes with the Witten's geometry [12], the chiral symmetry is spontaneously broken by adjoining the D8/$\overline{\text{D8}}$-branes there, as in the Sakai-Sugimoto model but with the technicolor instead of the QCD gauge group. This breaking will be mediated to the QCD sector of the N_c D4-branes by, for example, the massive gauge bosons coming from the open strings stretched between the D4 and the D4$'$-branes. This induces the bare masses for the QCD quarks.

From the Feynman graphs that would induce quark masses from a techni-quark condensate, we can easily identify the corresponding string worldsheets that mediate this phenomenon in our D-brane configuration in flat space. We consider a circle following D4-D8-D4$'$-$\overline{\text{D8}}$-branes (see figure 5) and a disk worldsheet instanton whose boundary is ending on this circle. It is responsible for the extended technicolor interactions coupling the quarks to the techni-quarks. Then we consider the strongly coupled field theory, or equivalently a weakly coupled gravity description, where we can identify the worldsheet instanton which has been responsible for the quark mass term, as we will study in section 2. In the gravity description, where both D4 and D4$'$-branes are replaced by a near horizon geometry of a multi-center gravity solution, we have a closed loop on the surface of the probe D8-branes only, since the D8- and the $\overline{\text{D8}}$-branes are smoothly connected at two throats created by the D4- and the D4$'$-brane geometry (see figure 7). The previous worldsheet instanton in the weak coupling picture is now ending on this closed loop. The worldsheet instanton is a leading order contribution to the effective action of the D8-branes, since it is a planar diagram with a single boundary in the large N gauge theory [13]. We will show that this worldsheet instanton amplitude indeed induces the lowest mass perturbation for pions that is expected in the standard low energy chiral Lagrangian. We will also show that the GOR relation is satisfied. This will be presented in detail in section 2.

Since the chiral symmetry is broken spontaneously from the viewpoint of the whole GUT theory (QCD and the technicolor is unified there), there should still exist massless Goldstone bosons. These modes will be localized around D4$'$ throat when the techni-quark condensate scale is much bigger than the QCD scale. In the gravity picture, they would correspond to the Wilson line associated with a path (on the D8-branes) coming from the asymptotic boundary to the D4$'$ throat and again back to the boundary. We have to take a limit to decouple these Nambu-Goldstone bosons by scaling $N'\to\infty$ first and putting D4$'$-branes at far UV region from the D4-branes, while keeping finite the mass of pions associated with the chiral symmetry breaking realized by the N_c D4-branes. This limit is difficult to achieve. In addition, it is cumbersome to obtain the multi-center solutions of D4-branes explicitly. Furthermore, realizing flavor-dependent quark mass in this set-up is not so easy.

To overcome these difficulties residing in the technicolor model, we then consider introducing N' D6-branes instead of the D4$'$-branes, which is free from these difficulties. The new deformation of the Sakai-Sugimoto model, the introduction of the D6-branes, not only keeps the essential idea of the previous idea of introducing the N' D4-branes, but also has great advantages of computability and unnecessity of taking any subtle limits. Therefore, we can compute the quark mass as well as the pion mass more rigorously, and consequently we can estimate the value of the chiral condensate of the Sakai-Sugimoto model via the GOR relation. We will also see that the numerical values of the quark masses derived from our formulae, with the pion mass as an input, are around 6 MeV, which happens to be quite close to the real QCD. Our numerical value of the chiral condensate is close to the results of lattice QCD. These will be presented in section 3.

In the next section, we first give the field theoretical interpretation of introducing the N' D4-branes and describe the Feynman graphs which induce the quark masses. Then we study the corresponding string worldsheet instantons both in the D-brane configuration in flat space and in the holographic dual gravity description, and estimate the quark and the pion masses. After explaining difficulties of precise computations, in section 3 we instead introduce the N' D6-branes and study the corresponding interpretation in QCD. We carry out explicit computations of the quark and the pion masses. We show in detail that the up and down quarks have masses around 6 MeV in our model. It is important to stress that we can also realize flavor-dependent quark masses with our D6-branes. Since our idea has a field theoretical interpretation which should be universal among a wide variety of holographic QCD models, we apply our idea to a holographic model of QCD proposed by Kruczenski et al. [14] in section 4. We give a detail about how our idea works in their model. In section 5, we conclude with discussions.

2. Quark mass deformation and worldsheet instantons

In the Sakai-Sugimoto model with N_c D4, N_f D8 and N_f $\overline{\text{D8}}$-branes, QCD gauge bosons live on the D4-branes and the chiral flavor symmetry $\mathrm{U}(N_f)_L \times \mathrm{U}(N_f)_R$ is realized as a gauge symmetry on the N_f D8- and N_f $\overline{\text{D8}}$-branes. The left (right) handed massless quarks are localized at the intersection of the D4-branes and the D8- ($\overline{\text{D8}}$-) branes. In the holographic dual description, the near horizon geometry of the D4-branes terminates the spacetime at a certain radius, which naturally explains the QCD confinement. The chiral symmetry breaking in QCD is nicely realized, as the D8- and the $\overline{\text{D8}}$-branes are connected smoothly to each other at the tip of the geometry.

If we introduce additional D4-branes (which we call D4$'$-branes) which are parallel to but separated from the original D4-branes, the chiral symmetry is broken also by the D4$'$-branes in the holographic dual description. This represents the strong technicolor dynamics of the D4$'$-branes. Since there are massive gauge bosons coming from excitations on the open strings stretching between the D4- and D4$'$-branes, the effect of the chiral symmetry

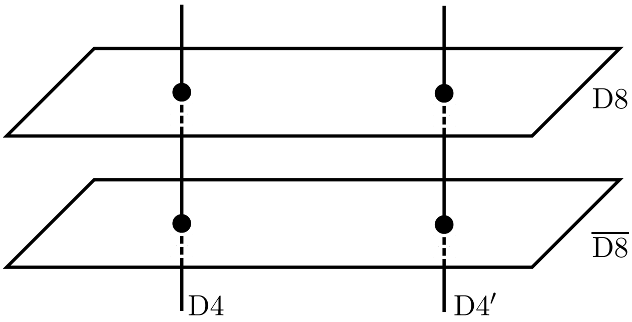

Figure 1: The D-brane configuration

breaking at the D4′-branes is transmitted to the QCD sector on the D4-branes, and the quark mass terms are resultantly induced. We will pursue this idea in this section.

2.1 Quark mass deformation

We first describe the D-brane configuration and study the low energy theory on the D-branes. The D-brane configuration which we now consider consists of D4, D8 and $\overline{\text{D8}}$-branes (see figure 1) and is summarized in the table:

	0	1	2	3	4	5	6	7	8	9
N_c D4	○	○	○	○	○					
N_f D8/$\overline{\text{D8}}$	○	○	○	○		○	○	○	○	○
N' D4′	○	○	○	○	○					

N_c D4-branes are localized at $x^5 = \cdots = x^9 = 0$ and N' D4-branes are localized at $x^5 \neq 0$ and $x^6 = \cdots = x^9 = 0$. The D8 and $\overline{\text{D8}}$-branes are localized at $x^4 = 0$ and $x^4 = \delta\tau/2$, where x^4 direction is compactified by a supersymmetry breaking S^1 with the periodicity $x^4 \sim x^4 + \delta\tau$. The compactification scale determines the scale of this system. This unique scale in the Sakai-Sugimoto model [2] is $M_{\text{KK}} \equiv 2\pi/\delta\tau \sim 1\,\text{GeV}$.

The N' D4-branes realize another massless QCD with SU(N') gauge symmetry as a low energy effective theory, in addition to the massless QCD with SU(N_c) of the Sakai-Sugimoto model. Thus the massless modes are SU(N_c)$\times$SU(N') gauge bosons and the two sets of left handed and right handed chiral massless quarks (q_L, q_R) and (Q_L, Q_R) which are fundamentally charged under SU(N_c) and SU(N') respectively. The massless modes are summarized in the following table:

	$SU(N_c)$	$SU(N')$	$U(N_f)_L$	$U(N_f)_R$
A_μ	adj.			
q_L	□		□	
q_R	□			□
A'_μ		adj.		
Q_L		□	□	
Q_R		□		□

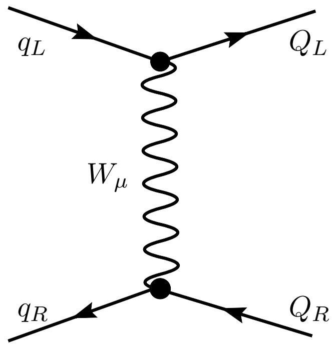

Figure 2: The four-Fermi interaction mediated by massive gauge bosons W_μ.

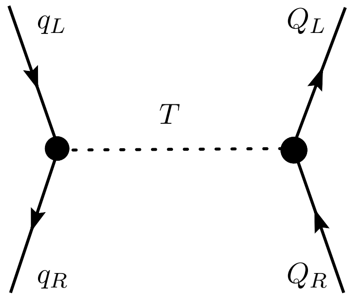

Figure 3: The four-Fermi interaction mediated by a massive scalar field T.

Since Q_L (Q_R) come from the open strings stretching between N' D4 and N_f D8 ($\overline{\text{D8}}$)-branes, the flavor symmetry is still $\mathrm{U}(N_f)_L \times \mathrm{U}(N_f)_R$. And there are massive modes which connect (q_L, q_R) and (Q_L, Q_R). They are the modes originated from the open strings stretching between the D4- and the D4$'$-branes or the D8- and the $\overline{\text{D8}}$-branes. The lightest modes from these two strings are massive gauge bosons W_μ and complex scalar field T whose charges are summarized in the table below:

	$SU(N_c)$	$SU(N')$	$U(N_f)_L$	$U(N_f)_R$
W_μ	$\square$	$\square$		
T			$\square$	$\square$

Their masses are computed from the distance between the D4- and the D4$'$-branes or the D8- and the $\overline{\text{D8}}$-branes.[1] The quarks have a gauge and Yukawa interaction with W_μ and T fields. So there are two Feynman graphs (shown in figure 2–3) which generate a four-Fermi interaction $\bar{q}_L q_R \bar{Q}_L Q_R$+h.c. mediated by the massive gauge bosons W_μ or the scalar field T.

At low energy, both $\mathrm{SU}(N_c)$ and $\mathrm{SU}(N')$ gauge symmetries become confined and the chiral symmetry is broken by condensates $\langle \bar{q}_L q_R \rangle$ and $\langle \bar{Q}_L Q_R \rangle$. By picking up a conden-

[1] The field T has been referred to as a "tachyon" since it is from a string connecting the D8 and the $\overline{\text{D8}}$-branes. However, when those branes are separated as in the present case, it is massive. In the low energy limit $l_s \to 0$ on the D4-branes, the mass of T diverges.

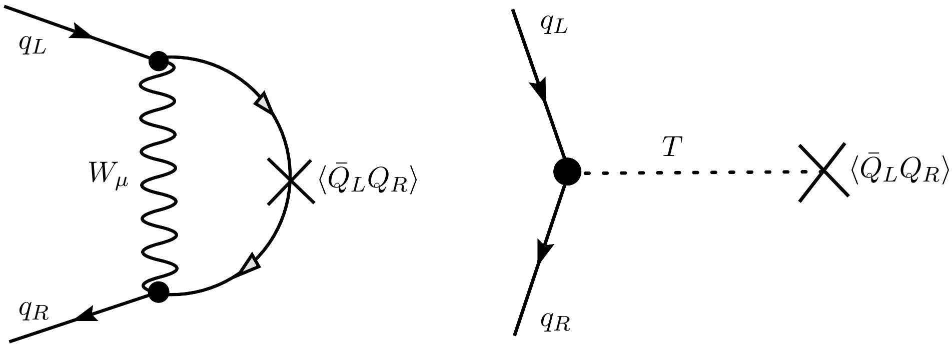

Figure 4: The quark mass is generated from the condensate $\langle \bar{Q}_L Q_R \rangle$.

sate $\langle \bar{Q}_L Q_R \rangle$, we observe from the four-Fermi interaction that the quark (q_L, q_R) becomes massive $m_q \propto \langle \bar{Q}_L Q_R \rangle$. See figure 4. This mechanism is quite similar to the one inducing quark masses in extended technicolor theories.

From the low energy effective field theory on the D4- and the D4′-branes in the weak coupling picture, we can estimate the quark masses m_q. The effective theory on the D4D4′-branes is essentially a five dimensional theory, in which quarks are localized at spatially separated 4 dimensional surfaces where they intersect with the D8-branes. Thus the calculation involves Kaluza-Klein (KK) reduction along the S^1. This kind of calculations has been done in the context of brane world scenario in [15] (see appendix A for our relevant calculations). We just mention that the amplitude in figure 2 is suppressed by $\exp[-\pi M_W/M_{\rm KK}]$ since the W-boson with the mass M_W must propagate from $\tau = 0$ to $\tau = \delta\tau/2 = \pi/M_{\rm KK}$.

We can similarly compute the Feynman graph figure 3 and obtain the same suppression $\exp[-\pi M_W/M_{\rm KK}]$: the D8/$\overline{\rm D8}$-brane tachyon T with its mass $\frac{1}{2\pi\alpha'}\frac{\delta\tau}{2}$ propagates horizontally from the D4′ to the D4 by a distance $U_0 = 2\pi\alpha' M_W$. In string theory, the string worldsheet which represents these two graphs is shown to be identical. An *st*-channel duality on the string worldsheet is manifested in these two Feynman graphs. In fact, as we will see, estimation of this suppression factor is easier, if we study the corresponding string worldsheets.

2.2 Worldsheet instanton

The string worldsheets which correspond to the Feynman graphs in figure 2 and figure 3 can be easily identified. The chiral quarks are localized on the intersections of D-branes of different kinds (i.e. the D4-branes and the D8-branes), and open strings stretching between D-branes of the same species (i.e. between the D4- and the D4′-branes and between the D8-branes) induce the massive gauge bosons or the scalar field. In addition, the Feynman graphs in figure 2–3 are tree graphs, so we find that the topology of the corresponding string worldsheets is disk, and the boundary of the worldsheet is on a closed loop surrounded by D4-D8-D4′-$\overline{\rm D8}$ branes. See figure 5 in which the worldsheet is shown by a shaded region. The worldsheet boundary has four boundary twist vertex operators at the corners which are

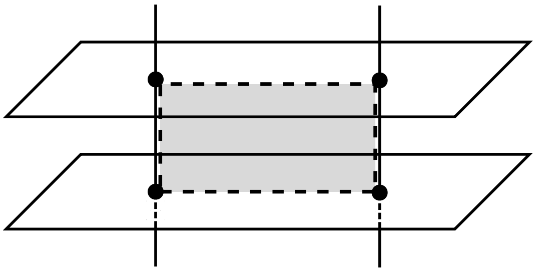

Figure 5: The shaded region denotes a worldsheet instanton. A dotted line is a closed loop surrounded by the D4-D8-D4′-$\overline{\mathrm{D8}}$ branes.

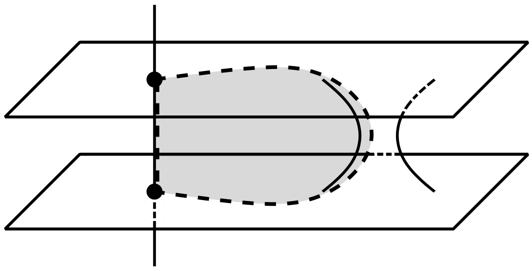

Figure 6: The shaded region denotes a worldsheet instanton. The throat is developed at the location of D4′-branes in the geometry.

the intersections of the D-branes. Since these worldsheets are localized in time direction, they are worldsheet instantons.[2]

The quark masses are induced by the chiral condensate $\langle \bar{Q}_L Q_R \rangle$ realized by strongly interacting SU(N') gauge dynamics. This dynamics can be captured from a weakly coupled holographic dual description by replacing the D4′-branes by the near horizon geometry. (Note that the D4-branes are still probes in order for the SU(N_c) sector to be in a weak coupling regime.) The geometry is given by Witten [12] in which the spacetime is terminated at the region centered at the D4′-branes. This forces the D8- and the $\overline{\mathrm{D8}}$-branes to connect with each other smoothly, which is a realization of the spontaneous chiral symmetry breaking.

In this geometry, the worldsheet instanton which we evaluate is shown in figure 6. The boundary of the worldsheet is on the D4- and the D8-branes with two twist vertex operators corresponding to the quarks. We can show that the quarks become massive due

[2] When we have $N_f > 1$ number of D8- and $\overline{\mathrm{D8}}$-branes, we have other worldsheets ending on, for example, D4$^{(i)}$-D8-D4′$^{(j)}$-D8 branes. These worldsheets do not give rise to the quark masses (and the pion masses), because the boundary needs vertex operators which changes the Cartan charges of the flavor group. See section 3.4 for detailed discussions.

to the instanton amplitude evaluated in a saddle point approximation[3]

$$S_{\rm instanton} = c \int d^4x \ {\rm tr} \ \bar{q}_L q_R \exp[-S_{\rm NG}] + {\rm h.c.}. \tag{2.1}$$

Here the trace is taken over the flavor $\mathrm{U}(N_f)$ indices and $S_{\rm NG}$ is the classical value of the Nambu-Goto action. The Hermitian conjugate comes from the oppositely oriented worldsheets, and c is a constant factor. We have to evaluate $S_{\rm NG}$ in the curved geometry of [12] which is

$$ds^2 = \left(\frac{U}{R'}\right)^{3/2} \left(dx_4^2 + f(U) d\tau^2\right) + \left(\frac{R'}{U}\right)^{3/2} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right), \tag{2.2}$$

$$e^{\phi} = g_s \left(\frac{U}{R'}\right)^{3/4}, \quad F_4 = \frac{2\pi N'}{V_4}\epsilon_4, \quad f(U) = 1 - \frac{U'^3_{\rm KK}}{U^3}, \tag{2.3}$$

$$R'^3 = \pi g_s N' l_s^3, \qquad M_{\rm KK} = \frac{3U'^{\frac{1}{2}}_{\rm KK}}{2R'^{\frac{3}{2}}}, \tag{2.4}$$

where $U \geq U'_{\rm KK}$ is the radial direction transverse to the D4$'$-branes, g_s is the string coupling, $l_s = \sqrt{\alpha'}$ is the string length, V_4 and ϵ_4 are the volume and line element of S_4.

Let the probe N_c D4-brane be placed at $U = U_0$. Since the minimal size worldsheet instanton extends along U and τ directions, we have

$$S_{\rm NG} = \frac{1}{2\pi\alpha'} \int_0^{\delta\tau/2} d\tau \int_{U'_{\rm KK}}^{U_0} dU \sqrt{g_{\tau\tau} g_{\rm UU}} = \frac{U_0 - U'_{\rm KK}}{2\alpha' M_{\rm KK}} = \frac{\pi M_W}{M_{\rm KK}} \tag{2.5}$$

in the approximation $U_0 \gg U'_{\rm KK}$, where M_W is the mass of the massive gauge boson W_μ computed as $M_W = U_0/(2\pi\alpha')$. We finally obtain

$$m_q = c \exp\left[-\frac{\pi M_W}{M_{\rm KK}}\right]. \tag{2.6}$$

In appendix A, we evaluate the four-Fermi interaction in field theory perturbation and obtain the same suppression factor. This fact supports our identification of the worldsheet instantons.[4]

[3]The worldsheet instanton amplitude (2.1) is similar to what is popularly used in string phenomenology (see for example [16]). In D-brane construction of the standard model in string theory, one puts various D-branes intersecting with each other in higher dimensional compact space. Then the standard model fields appear on the intersections. To compute Yukawa couplings of those fields, worldsheet instantons are used. The magnitude of the Yukawa coupling is proportional to $\exp[-S_{\rm NG}]$ where $S_{\rm NG}$ is a classical part of the Nambu-Goto action for the Euclidean worldsheet spanning the triangle made of the intersections relevant for the fields composing the Yukawa interaction. We apply here the same idea to the worldsheet shown in figure 6. Since we have two intersections, we insert string vertex operators on those intersections, which become the quark mass term.

[4]However, note that the field theory calculation in appendix A is based on a tree graph in a field theory perturbation and so it is valid only for small coupling constant for both the QCD gauge group and the technicolor gauge group. Here in this subsection to derive (2.6), the D4$'$ technicolor sector is strongly coupled, therefore we needed to compute the quark mass term by using the AdS/CFT correspondence at the technicolor sector. So, the quark mass (A.8) obtained in the field theory perturbation in appendix A is not equal to (2.6). The calculations presented in appendix A are just for an illustration at weak coupling in the technicolor sector. Note that the gauge coupling at the QCD sector is still weak in both cases.

JHEP07(2008)089

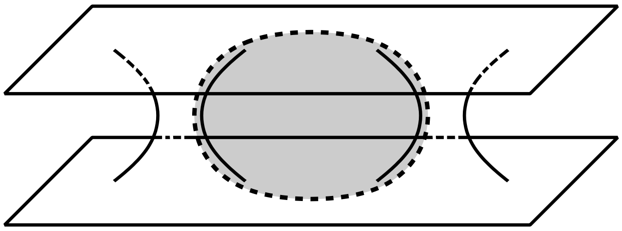

Figure 7: The shaded region denotes a worldsheet instanton. The two throats are developed at the location of D4 and D4′-branes.

The constant factor c in (2.6) depends on the string length, the string coupling and in particular the chiral condensate $\langle \bar{Q}_L Q_R \rangle$, so it is an important coefficient. However it is difficult to compute c in the curved geometry, so we do not elaborate on the evaluation of c. This leads us to the study of introducing D6-branes instead of the D4′-branes in the next section. There, with a different configuration of D-branes, we can compute the quark mass in a more definite manner.

2.3 Pion mass

We found that the quark becomes massive due to the chiral symmetry breaking caused by the N' D4-branes. Thus we should be able to see that the pions become massive and the mass should satisfy the GOR relation.

In the low energy limit, both SU(N_c) and SU(N') become strongly coupled. There, by the AdS/CFT correspondence, a holographic dual description gives a weakly coupled description in terms of gauge invariant operators, i.e. mesons. For obtaining the gravity dual, we need a multi-center solution of the D4-branes and its near horizon geometry, because now we have the D4′-branes in addition. However, any explicit metric for it is not known. Thus in this subsection we give only a qualitative discussion on how the pions become massive in the holographic dual description. In the next section we study another model in which quantitative calculations are possible.

The geometry of the two-center solution has two "throats" at the location of the D4- and the D4′-branes respectively. See figure 7. The geometry should be obtained by gluing two metrics (2.2). Thus we again have the worldsheet instantons denoted by the shaded region in the same figure. The amplitude of this worldsheet instantons is given by

$$S_{\text{instanton}} \propto \int d^4x \frac{1}{g_s} \mathrm{P}\mathrm{tr}\exp\left[-S_{\text{NG}} + i\oint dz A_z\right] + \text{h.c.}, \tag{2.7}$$

where we have included a standard worldsheet boundary coupling to A_z, and z parameterizes the boundary of the worldsheet instanton. The factor $1/g_s$ is introduced because the worldsheet is a disk. Since the pion wave function is localized at the D4 throat [2], we have

$$\mathrm{P}\mathrm{tr}\exp\left[i\oint dzA_z\right] = \mathrm{tr}\, U, \quad U \equiv \exp\left[2i\pi(x)/f_\pi\right], \tag{2.8}$$

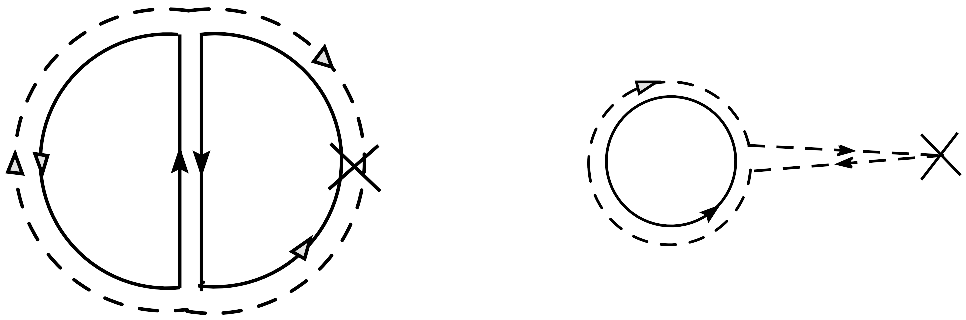

Figure 8: The Feynman diagrams responsible for quark masses can be written in the double-line notation, which shows that the flavor line (dashed lines) form a closed single loop. Solid lines are for the color indices.

after substituting into A_z the pion wave function. Then we obtain

$$S_{\text{instanton}} = m_\pi^2 f_\pi^2 \,\text{tr}(U + U^\dagger), \quad m_\pi^2 \propto \frac{1}{g_s} \exp\left[-S_{\text{NG}}\right] \propto m_q \tag{2.9}$$

because the exponential factor $\exp[-S_{\text{NG}}]$ is roughly equal to that of the quark mass. We find that the GOR relation is naturally derived. From the AdS/CFT, we find here that, in the pion chiral Lagrangian, the quark mass perturbation gives an additional term (2.9). This (2.9) turns out to be the well-known standard pion mass term in chiral perturbation theory.

Before closing this section, we give some interesting observations. The holographic description is reliable in the large N_c (and N') and large 't Hooft coupling, and a genus zero string worldsheet corresponds to a planar diagram in a large N QCD, a la 't Hooft. The Feynman graphs using the double-line notation in a large N QCD is shown in figure 8. It is clear from this that these Feynman diagrams are included in a planar diagram with a single boundary. On the other hand, the worldsheet instanton is a disk amplitude which is nothing but a planar diagram with a single boundary. This argument presents a consistency for our identification of the four-Fermi Feynman graphs with the worldsheet instantons.

The worldsheet instanton does not contribute to the Dirac-Born-Infeld (DBI) action S_{DBI} for the D8-branes, i.e. the total action S_{D8} is

$$S_{\text{D8}} = S_{\text{DBI}} + S_{\text{instanton}}. \tag{2.10}$$

There must not be a mass term for the pions from S_{DBI} at the leading order in large N. We can easily see this from the fact that the pion is in a KK zero mode of a gauge field A_z on the D8-branes. In the DBI action, the gauge fields always appear as their field strengths. Thus the pion $\pi(x)$ should be accompanied by four-dimensional derivatives, i.e. $\partial_\mu \pi(x)$, and cannot have a mass term in the DBI action.

In this section, we focused on the quark (q_L, q_R) and the pion in the $\text{SU}(N_c)$ sector. However, our D4$'$-branes realize another $\text{SU}(N')$ gauge theory with the techni-quarks (Q_L, Q_R) and so there is another pion in the $\text{SU}(N')$ sector. In total, the chiral symmetry is still spontaneously broken, one combination of the two pions is still a massless

Nambu-Goldstone boson. In order for the massless mode to be decoupled from the QCD sector, we need to take a certain limit. The desired limit should be the following: the pion decay constant[5] for this unnecessary pion is taken to be infinite by $N' \to \infty$, as well as the distance from D4-branes taken to be infinite while the pion mass we computed kept finite. Concrete realization of this limit is not clear to us, because we can not explicitly compute the coefficient of the instanton amplitudes.

Therefore, in the next section, we will consider another deformation of instead introducing D6-branes. This deformation still have the strategy in this section, while it needs no complicated limit: the chiral symmetry is explicitly broken by the D6-branes.

3. Quark mass deformation by D6-branes and pion mass

In this section, we propose another way to introduce quark masses to the Sakai-Sugimoto model. In the approach of this section, the evaluation of the quark and pion mass terms is more tractable. First we explain the idea of introducing a probe D6-brane ending on the D8- and the $\overline{\text{D8}}$-branes, instead of introducing the D4$'$-branes. Then, we compute the worldsheet instanton explicitly to obtain the quark masses, pion mass and the chiral condensate. Finally, we present a numerical evaluation of those values, for a tentative comparison with experimental or lattice QCD's results.

3.1 Configuration of the D6 ending on D8 and $\overline{\text{D8}}$

As seen in the previous section we can introduce the quark mass term if we have a 1-cycle in the brane configuration, because the worldsheet instantons are possible. Once we stand on this point of view, it is obvious that we need not insist in using the D4$'$-branes. Instead, we introduce N' D6-branes which are separated away from the D4-branes and end on the D8-branes and the $\overline{\text{D8}}$-branes. Each D6-brane can end on different D8- and $\overline{\text{D8}}$-branes. The orientation of the additional probe D6-brane is shown in the table.[6]

	0	1	2	3	4	5	6	7	8	9
N_c D4	○	○	○	○	○					
N_f D8 $\overline{\text{D8}}$	○	○	○	○		○	○	○	○	○
N_f D6	○	○	○	○	○		○	○		

This brane-ending-on-brane configuration is represented by a stable spike solution on the D8-branes. This means that the D8-branes and the $\overline{\text{D8}}$-branes are smoothly connected with each other by the smooth spike (volcano-shaped) [17], and thus there is no twist operator insertion at the end points of the D6. We can obtain a worldsheet instanton whose shape is almost the same as that of the case introducing the D4$'$-branes.[7]

[5] The pion decay constant is computed in [2]and it is proportional to $\sqrt{N'}$ for fixed 'tHooft coupling. Therefore, if one takes the limit $N' \to \infty$, this realizes an effective decoupling of the pion.

[6] To have a stable configuration of the D6-brane, the D6-brane is curved when the D4-brane is replaced by its geometry. In the appendix B, we find a consistent curved shape of the D6-brane in the background, by solving the equations of motion of the D6-brane effective action.

[7] This instanton is similar to the one considered in [18] where QCD instanton is studied. We thank J. Sonnenschein for pointing this out.

Let us summarize our field theoretical understanding on the D6-branes with emphasis on the differences from the D4′-brane case.

- The D6-brane can be treated as a probe ($N' \ll N_c$), hence it does not modify the background geometry. The D6-brane is a classical solution of the effective theory of the flavor D8-branes (or the $\overline{\text{D8}}$-branes), and it is a part of the probe flavor D-branes. Thus, on the contrary to the previous section, the background geometry is still given by just the N_c D4-branes i.e. Witten's geometry of a single center.

- The introduction of the D6-brane breaks the chiral symmetry, since the D6-branes end on the D8 and $\overline{\text{D8}}$-branes and thus connect the D8 and $\overline{\text{D8}}$-branes. The breaking is explicit, not spontaneous, since the D6-branes extend along (x^6, x^7) infinitely and the fields localized on the D6-branes, which would be Nambu-Goldstone bosons, do not have a finite kinetic term in the four dimensions.

- From the theory on the D8-branes, the D6-branes are interpreted as monopoles with the charge $(1, -1)$ under the chiral symmetry for a single flavor case $\mathrm{U}(1)_L \times \mathrm{U}(1)_R$. Thus the source for the Dirac monopole explicitly breaks the axial symmetry. For D6-branes not on top of each other, all the axial symmetry is broken, which corresponds to the situation where quark masses are not coincident with each other.

- WE have essentially the same field theoretical understanding of how the effects of the chiral symmetry breaking on the D6-brane sector are transmitted to the QCD sector on the D4-branes. Here we have massive scalar fields (instead of massive gauge bosons) from the open strings between the D4-branes and the D6-branes which, for example, play the role of the mediators.

- We have an easy way to realize flavor-dependent quark masses by shifting the location of each D6-brane independently. Another way (which can also apply for the D4′-branes) is to shift the D8-brane from the anti-podal points, but the D8-brane configuration away from the anti-podal point is known only numerically [19], which makes the estimate more difficult.

3.2 Quark mass from worldsheet instanton

Let us first compute the quark mass term. At this stage we keep the D4-branes to be probes, that is, the spacetime is still flat, because the quark mass term can be read in QCD Lagrangian at weak coupling. (In the next subsection we replace them with a curved geometry.) The mass generation mechanism by the worldsheet instantons is quite the same as in the previous section, see figure 6. The throat on the right hand side of the figure is now N' spiky D6-branes ending on the D8-branes and the $\overline{\text{D8}}$-branes. At the end points of the D6-brane, there is no insertion of a vertex operator because the D6 and the D8 join smoothly there. Resultantly, the worldsheet instanton amplitude should be given by

$$\frac{\mathcal{N} N'}{g_s l_s} e^{-S_{\mathrm{NG}}} \int d^4x \; \bar{q}_{\mathrm{L}} q_{\mathrm{R}}. \tag{3.1}$$

The derivation of the worldsheet instanton amplitude (3.1) is based on a standard formula which has been used extensively in computations of Yukawa couplings in D-brane construction in string phenomenology [16]. The amplitude is suppressed by the exponential factor whose exponent is the classical part of the Nambu-Goto action for the worldsheet spanning the Euclidean area encircled by the D4-D8-D6-$\overline{\text{D8}}$. The Nambu-Goto worldsheet action $S_{\rm NG}$ in (3.1) is just the area of the worldsheet times string tension in flat space, so we have $S_{\rm NG} = (1/(2\pi\alpha'))U_0(\pi/M_{\rm KK}) = \pi M_W/M_{\rm KK}$ where U_0 is the length of an open string stretching between the D4- and D6-branes and then M_W is the mass of modes from the open string. The factor N' in (3.1) comes in since the boundary of worldsheet instantons can be on N' different D6-branes. The front factor $1/g_s$ is for a disk amplitude, and the $1/l_s$ factor should be provided from the normalization of the quark vertex operator, to make a sensible dimensionality of the amplitude. The dimensionless number $\mathcal{N}$ is a normalization factor from the quark vertex operators, the quark kinetic terms and the fluctuations around our semi-classical worldsheet.

Note that the expression (3.1) is derived in completely flat background geometry, as there is no technicolor D4'-brane (which modifies the background geometry as in the derivation of (2.6)) but now we have the probe D6-branes instead.[8] For the technicolor models in the previous section, one needs to go to the strongly coupled regime for the technicolor sector, thus AdS/CFT is necessary for the computation of the technicolor sector, that is a complication for deriving the coefficient c. Here, we work in the flat spacetime, thus the computation of the worldsheet instanton, (3.1), is much clearer. The formula (3.1) is valid at small g_s as we are using string perturbation.

For the multi-flavor case, by shifting the position of each D6-brane respectively, we can introduce quark masses depending on flavors. For each flavor labeled by i, we can associate the location $U = U_0^{(i)}$ of coincident N' D6-branes (then the total number of the D6-branes is $N'N_f$). Then obviously, the quark mass term is a sum of each instanton, $\sum_i \int d^4x\, \bar{q}_{\rm L}^{(i)}(m_q)_{ij} q_{\rm R}^{(j)}$, with the following diagonal quark mass matrix

$$(m_q)_{ij} = \mathcal{N} N' \delta_{ij} \frac{2\pi M_{\rm KK}}{g_{\rm YM}^2} \exp\left[-\frac{\pi M_W^{(i)}}{M_{\rm KK}}\right], \tag{3.2}$$

where $M_W^{(i)} = U_0^{(i)}/(2\pi\alpha')$. The quark mass formula (3.2) is valid at small $g_{\rm YM}$ and for $M_W > M_{\rm KK}$.

3.3 Pion mass, GOR relation and chiral condensate

Next, we study the low energy limit in the holographic dual by replacing the D4-branes by their geometry (Witten's geometry) and compute the pion mass term. Since there is a nontrivial 1-cycle on the D8-brane, the worldsheet instanton gives us a pion mass term (see figure 7). The mechanism is completely analogous to the previous section. In the following, we present explicit evaluation of the worldsheet instanton amplitude. We will adopt some crude approximations for obtaining the results because a curved background makes the explicit evaluation still difficult.

[8] Therefore, the coefficient c in (2.6) is not related to (3.1).

The worldsheet instanton amplitude is given as

$$S_{\text{instanton}} = N' \frac{1}{\tilde{g}_s} \frac{1}{(2\pi)^3 l_s^4} \int \sqrt{-\det g}\, d^4x\, e^{-S_{\text{NG}}} \left[\text{Ptr} \exp \left[-i \oint A_z dz \right] + \text{c.c.} \right], \tag{3.3}$$

where the trace is taken over the flavor indices. To obtain this expression, we made the following approximations as we will explain step by step.

The classical worldsheet action S_{NG} is evaluated in the same way computed in the previous section, i.e., see (2.5), so $S_{\text{NG}} = \pi M_W / M_{\text{KK}}$.

Let us explain the front factor $1/(\tilde{g}_s (2\pi)^3 l_s^4)$ in (3.3). First, we notice that the worldsheet instanton is wrapping the minimal cycle whose zero mode is 4-dimensional. The worldsheet instanton cannot move along the direction of the S^4: the boundary should be on the D6-brane whose closest point to the D4-branes is just a point on the S^4 (see appendix B for details). So the instanton amplitude should be proportional to a D3-brane tension $\mathcal{T}_{\text{D3}}$, not $\mathcal{T}_{\text{D8}}$.[9] The D3-brane tension is given by the front factor $1/(\tilde{g}_s (2\pi)^3 l_s^4)$.

Note that the string coupling constant should be the effective coupling constant $\tilde{g}_s$ obtained by including the effect of the background dilaton, $1/\tilde{g}_s = (1/g_s)\,(R/U_{\text{KK}})^{3/4}$ (see the classical solution (2.2) but remove the prime in the corresponding quantities). In getting this expression we made a crude (but reasonable) approximation that the U-dependence of the dilaton can be approximated by the value at $U \sim U_{\text{KK}}$. This is because the pion wavefunction is localized around U_{KK} so that most of the contribution for the integral over U would come from the region $U \sim U_{\text{KK}}$, even though the worldsheet itself is elongated in the region $U_{\text{KK}} < U < U_0$. Furthermore, the invariant volume $\sqrt{\det g}$ in the (x^0, x^1, x^2, x^3) directions was inserted in (3.3) for consistency. Using the metric expression (2.2), the four-volume is given as $\sqrt{-\det g} = (U_{\text{KK}}/R)^3$.

We can calculate the instanton amplitude (3.3) using the relations between the gravity and the gauge theory quantities, namely,

$$R^3 = \frac{g_{\text{YM}}^2 N_c l_s^2}{2 M_{\text{KK}}}, \quad U_{\text{KK}} = \frac{2}{9} g_{\text{YM}}^2 N_c M_{\text{KK}} l_s^2, \quad g_s = \frac{g_{\text{YM}}^2}{2\pi M_{\text{KK}} l_s}, \tag{3.4}$$

then we arrive at

$$S_{\text{instanton}} = \frac{2N'}{3^{9/2}\pi^2} g_{\text{YM}} N_c^{3/2} M_{\text{KK}}^4 \exp\left[-\frac{\pi M_W}{M_{\text{KK}}} \right] \int d^4x\, (\text{tr} U + \text{tr} U^\dagger). \tag{3.5}$$

This is the pion mass term generated by the worldsheet instanton in the curved background. Again, we found an expression well-known in chiral perturbation theory. Substituting the quark mass formula (3.2), this can be re-written as

$$S_{\text{instanton}} = \frac{1}{3^{9/2}\pi^3} g_{\text{YM}}^3 N_c^{3/2} M_{\text{KK}}^3 \mathcal{N}^{-1} m_q \int d^4x\, (\text{tr} U + \text{tr} U^\dagger). \tag{3.6}$$

[9] Along the S^4 directions transverse to the worldsheet instanton configuration, the string fluctuation follows Dirichlet boundary condition rather than Neumann. This effectively reduces the zero mode integral of the vertex insertion at the boundary of the worldsheet instanton. The number of the transverse directions is 4 in the worldvolume of the D8-branes. Taking into account the z direction along which the worldsheet boundary is elongated, we need to introduce the factor $\mathcal{T}_{\text{D3}}$ in front of the worldsheet instanton amplitude, rather than just $1/g_s$.

Expanding $U = \exp[2i\pi(x)/f_\pi]$ to the second order in the pion field $\pi(x)$, we obtain the GOR relation

$$m_\pi^2 = \frac{4g_{\rm YM}^3 N_c^{3/2} M_{\rm KK}^3}{3^{9/2}\pi^3 \mathcal{N}} \frac{1}{f_\pi^2} m_q. \tag{3.7}$$

The chiral condensate computed in our model is, therefore,

$$\langle \bar{q}q \rangle = \frac{2}{3^{9/2}\pi^3} \mathcal{N}^{-1} g_{\rm YM}^3 N_c^{3/2} M_{\rm KK}^3. \tag{3.8}$$

This is the expression for the chiral condensate in the Sakai-Sugimoto model. [10]

Note that, as we stated, we made crude approximations in evaluating the coordinate dependence of the background dilaton and metric. But it is nontrivial that all the l_s dependence disappears at the end, which may signal a consistency of the approximation.

3.4 Flavor-dependent quark masses and numerical results

In this subsection we consider the flavor dependent quark masses by placing the D6-branes at different points. The worldsheet instanton ends in part on the D6-brane, and let us consider the transverse scalar field Φ on the D6-branes. The separation of the D6-branes can be encoded in the worldsheet boundary interaction as a condensation of the transverse scalar field Φ. Because the D6-branes are made of the spike of the D8- and $\overline{\rm D8}$-branes, this Φ can be at the same time regarded as a transverse scalar field on the D8-$\overline{\rm D8}$-branes, too. So, in total, together with the usual boundary interaction for the gauge fields on the D8-branes, the boundary interaction is

$$\mathrm{Ptr}\exp\left[-i\oint A_z dz - \int_b \Phi dz\right] \tag{3.9}$$

The integration region b is a period (in the worldsheet boundary) where the worldsheet ends on the D6-branes.[11] We can parametrize the scalar field as $2\pi\alpha'\Phi = \mathrm{diag}(U_0^{(1)}, U_0^{(2)}, \dots)$ where $U_0^{(i)}$ denotes the location of the N' D6-branes[12] for the i-th flavor. If we neglect the dynamical fluctuation of the scalar field, then this is just a constant matrix.

The vertex insertion of the gauge field $A_z dz$ on the boundary of the worldsheet instanton is located almost around $U \sim U_{\rm KK}$ (due to the localized distribution of the pion wave function found in [2]), that is far away from the region where the transverse scalar Φ is inserted (where the D6-brane is present). Therefore, the path-ordering in (3.9) is effectively approximated by the following ordering

$$\mathrm{tr}\left[\mathrm{P}\exp\left[-\Phi\int_b dz\right]\cdot \mathrm{P}\exp\left[-i\oint A_z dz\right]\right]. \tag{3.10}$$

[10] An interesting feature of this chiral condensate is that, if $\mathcal{N} = 1$, it is independent of N_c if it is written with 'tHooft coupling $\lambda = g_{\rm YM}^2 N_c$. It sounds consistent with Coleman-Witten argument [20]. The N_c dependence of the chiral condensate found here is a little different from what was found in [14].

[11] We don't have the factor i in front of the scalar field coupling, because the scalar field is associated with the instanton which is Euclideanized, and we will see the consistency below.

[12] This is the location of the tip of the smeared D6-brane cone, see appendix B for the shape of the D6-brane. The tip is given by $\theta_1 = 0$.

Note that the integral $\int_b$ is performed only along the worldsheet boundary on the D6-branes. This means

$$\exp\left[-\Phi\int_b dz\right] = \exp\left[-\Phi\frac{\pi}{M_{\mathrm{KK}}}\right] = \exp\left[-\mathrm{diag}\left(\frac{\pi M_W^{(1)}}{M_{\mathrm{KK}}}, \frac{\pi M_W^{(2)}}{M_{\mathrm{KK}}}, \ldots\right)\right] = \frac{g_{\mathrm{YM}}^2}{2\pi M_{\mathrm{KK}}} m_q. \quad (3.11)$$

In this manner, we realize a non-Abelian generalization of the worldsheet area (previously written as $S_{\mathrm{NG}} \simeq \pi M_W/M_{\mathrm{KK}}$).[13] Using this expression, we obtain the following term induced by the worldsheet instanton:

$$S_{\mathrm{instanton}} = \frac{1}{3^{9/2}\pi^3} g_{\mathrm{YM}}^3 N_c^{3/2} M_{\mathrm{KK}}^3 \mathcal{N}^{-1} \int d^4x \; \mathrm{tr}\left[m_q(U + U^\dagger)\right]. \quad (3.12)$$

This form, $\mathrm{tr}[m_q(U+U^\dagger)]$, is in fact what is usually expected in chiral perturbation theory.

In the two-flavor case, we have $m_q = \mathrm{diag}(m_u, m_d)$, so

$$\mathrm{tr}\left[m_q(U + U^\dagger)\right] = 2(m_u + m_d)\left(1 - \frac{1}{f_\pi^2}\pi_a(x)^2 + \mathcal{O}(\pi(x)^4)\right), \quad (3.13)$$

where the component definition is $\pi(x) = \pi_a(x)\sigma_a/2$. So at this leading order estimate the mass of π^0 is equal to the mass of $\pi^\pm$, as in the usual chiral perturbation theory. Using the expression [2] for the pion decay constant $f_\pi = g_{\mathrm{YM}} N_c M_{\mathrm{KK}}/(3\sqrt{6}\pi^2)$, (3.12) gives a pion mass term

$$m_\pi^2 = 2(m_u + m_d)\frac{2\pi g_{\mathrm{YM}} M_{\mathrm{KK}}}{3\sqrt{3} N_c^{1/2}\mathcal{N}}. \quad (3.14)$$

The difference between π_0 and $\pi_\pm$ can be seen in higher-order chiral Lagrangian in chiral perturbation theory. In appendix C, we give a computation of two worldsheet instantons, to get the higher-order terms.

Just for an illustration, we present a numerical evaluation of the relation (3.7).[14] Sakai and Sugimoto [2] deduced numerical values as $g_{\mathrm{YM}} = 2.35, M_{\mathrm{KK}} = 949$ [MeV]. The first came from the equation $\kappa = \lambda N_c/(216\pi^3) = 0.00745$ with $N_c = 3$. Then, with $m_\pi = 140$ [MeV] as an input, the mass formula (3.14), with $\mathcal{N} = 1$ as an assumption, gives

$$m_u + m_d = 6.29 \text{ [MeV]}. \quad (3.15)$$

Experimental results shown in the particle data book are $m_u = 1.5 - 4.0$ [MeV], $m_d = 4 - 8$ [MeV]. Surprisingly, our result is very close to the observed value of the up/down quark mass.

[13] In the new notation here of (3.11), the classical part is $S_{\mathrm{NG}} = 0$. If instead one uses the minimal value of Φ as the common part of the action S_{NG}, then $S_{\mathrm{NG}} = \pi M_W^{(1)}/M_{\mathrm{KK}}$, and one should parameterize the transverse scalar field as its deviation from $U^{(1)}$, as $2\pi\alpha'\Phi = \mathrm{diag}(0, U_0^{(2)} - U^{(1)}, U_0^{(3)} - U_0^{(1)}, \ldots)$ to get the correct non-Abelian expression for the worldsheet boundary interaction.

[14] Readers are advised that the numerical values presented in this paragraph are just for illustration, and should not be taken seriously, as we made a crude approximation in evaluating the worldsheet instanton in the curved background, and we are working in the leading order in large N and large 'tHooft coupling expansion.

JHEP07(2008)089

The chiral condensate (3.8) is numerically evaluated as

$$\langle \bar{q}q \rangle = (299\ [\mathrm{MeV}])^3, \tag{3.16}$$

which is in quite good agreement with values obtained in quenched/unquenched lattice simulation, $\langle \bar{q}q \rangle \simeq (2.5 \times 10^2\ [\mathrm{MeV}])^3$.

4. Application to the holographic QCD of flavor D6-branes

Our idea is based on a field theoretical picture inspired by extended technicolor theories. The technicolor sector is coupled with the QCD sector via massive gauge-bosons of the extended gauge group, and the condensation of the techni-quarks Q gives rise to a quark mass term through the induced four-Fermi coupling. Therefore, this mechanism can be applied to other holographic models of QCD. In this section we demonstrate this by applying our idea to the D4-D6 model of Kruczenski et al. [14]. The model consists of N_c D4-branes giving rise to the curved geometry and N_f flavor D6-branes which are introduced as probes. In fact the model can describe massive quarks, as the D6-branes can be shifted away from the D4-branes. We will see that our technicolor D4′-branes have the same effect: they shift the D6-branes further and the quarks get extra masses because of this.

The D-brane configuration of the model in [14] consists of the D4 and D6-branes as in the table:

	0	1	2	3	4	5	6	7	8	9
N_c D4	○	○	○	○	○					
N_f D6	○	○	○	○		○	○	○		

The radial directions in (x^5, x^6, x^7) and in (x^8, x^9) from the D4-branes are denoted as λ and r respectively, and the D6-brane configuration in the D4-brane geometry is solved in their paper. The asymptotic behavior for $\lambda \gg \lambda_c$, (λ_c is some large value), is

$$r \sim r_\infty + \frac{c}{\lambda}, \tag{4.1}$$

where r_∞ is the asymptotic distance between the D4- and the D6-branes which is related to the bare quark mass m_q as

$$m_q = \frac{r_\infty}{2\pi l_s^2}, \tag{4.2}$$

and c is related to the value of condensate,[15]

$$\langle \bar{q}q \rangle \simeq \frac{g_{\mathrm{YM}}^2 N_c^2 M_{\mathrm{KK}}^3}{U_{\mathrm{KK}}^2} c \qquad \left(U_{\mathrm{KK}} = \frac{2}{9} g_{\mathrm{YM}}^2 N_c M_{\mathrm{KK}} l_s^2 \right) \tag{4.3}$$

The mass of the pion, which is a pseudo Nambu-Goldstone boson associated with the breaking of the rotation symmetry of the (x^8, x^9)-plane, is also computed in their paper,

$$m_\pi^2 \sim \frac{m_q M_{\mathrm{KK}}}{g_{\mathrm{YM}}^2 N_c}. \tag{4.4}$$

[15]The unit $U_{\mathrm{KK}} = 1$ is used in [14].

We now introduce our technicolor N' D4$'$-branes which are parallel to, but separated from the N_c D4-branes along λ with $r=0$ (so that $U=\lambda$). Let's say that the D4-branes are sitting at $\lambda=0$ and the D4$'$-branes are at $\lambda=\lambda_0$. We first treat the D4-branes as probes and consider the effects from confining D4$'$ dynamics. Assuming the distance $U_0=\lambda_0$ between the D4 and D4$'$-branes is much larger than λ_c, i.e. $U_0\gg\lambda_c$, the value of r at the position of the D4-brane $\lambda=0$ induced by the presence of D4$'$-brane would be

$$r\sim r_\infty+\frac{c'}{U_0}\quad , \tag{4.5}$$

where c' denotes the contribution from the D4$'$-branes, i.e.

$$\langle\bar{Q}Q\rangle\simeq\frac{g_{\rm YM}^2N'^2M_{\rm KK}^3}{U'^2_{\rm KK}}c'\qquad\left(U'_{\rm KK}=\frac{2}{9}g_{\rm YM}^2N'M_{\rm KK}l_s^2\right) \tag{4.6}$$

For the probe D4-branes in their weak coupling regime (that is, before their gravity background is considered), this additional shift (4.5) serves as its new asymptotic distance r_∞ from the $D6$ branes. It is shifted from the original r_∞ by $\frac{c'}{U_0}$. We then take the gravity background for our D4-branes, and we have

$$r\sim r_\infty+\frac{c'}{U_0}+\frac{c}{\lambda}, \tag{4.7}$$

for $\lambda_c\ll\lambda\ll U_0$.

Therefore we see that the quark mass in QCD sector on the D4-branes is shifted as

$$m_q\sim\frac{U_{\rm KK}}{2\pi l_s^2}\left(r_\infty+\frac{c'}{U_0}\right)=\frac{U_{\rm KK}r_\infty}{2\pi l_s^2}+\frac{g_{\rm YM}^2\langle\bar{Q}Q\rangle}{81\pi^2M_{\rm KK}M_W} \tag{4.8}$$

where we have used the W-boson mass $M_W=U_0/(2\pi l_s^2)$ of the D4-D4$'$ string. The mass of the pion in the QCD sector is read again

$$m_\pi^2\sim\frac{m_qM_{\rm KK}}{g_{\rm YM}^2N_c}. \tag{4.9}$$

In the above computation, we used the holographic dual description and used the fact that the pseudo Nambu-Goldstone wavefunction in QCD sector is localized around $\lambda=0$. On the other hand, we can also study the quark mass from a purely field theoretical point of view without using the AdS/CFT, since introducing the D4$'$-branes has an interpretation of introducing a techni gauge theory. The invariance under the rotation in the (x^8,x^9)-plane is broken by the D4$'$-branes and the breaking effects are transmitted to the QCD sector on the D4-branes by, for example, massive gauge bosons which come from the open strings connecting the D4- and D4$'$-branes. In the field theory perturbation, we then have a Feynman graph similar to figure 2 and can compute the four-Fermi coupling. We obtain the quark mass by replacing $\bar{Q}Q$ with its condensate $\langle\bar{Q}Q\rangle$ (this procedure is not really allowed in perturbation theory). After a careful calculation, the resultant quark mass computed is consistent with (4.8) up to a numerical factor.

A comment is in order. The D6-brane configuration is obtained by solving the equation of motion computed from the DBI action for the D6-brane in the D4-brane geometry. This means that the pion mass terms are induced in the DBI action because of the D4′-branes, on the contrary to the Sakai-Sugimoto model where the DBI action does not have a mass term for the pion. This is understood from the string world sheets. The string world sheet which corresponds to the graph in figure 2 is not a worldsheet instanton, but just a disk amplitude whose boundary is on the D6-branes. The minimal worldsheet area is vanishing. If the boundary passes through the two throats created by the D4 and D4′-branes, this disk worldsheet picks up the effects on the D4′-branes and communicate them into the D4-branes. Since the DBI action is computed from this infinitely small disk worldsheet (which is not a worldsheet instanton), the mass terms for the pion appear in the DBI action in the present case.

5. Conclusions and discussions

In this paper, we propose the deformations of the Sakai-Sugimoto model to generate the quark masses. Our considerations are motivated by extended technicolor theories where we break chiral symmetry via introducing a technicolor sector. We systematically trace the mass deformations in different descriptions, that is, weak coupling D-branes setting and the corresponding holographic gravity description. In the field theory side, the chiral condensate in the technicolor sector is mediated to the QCD sector, generating the quark masses. One then expects the massive quarks will yield the massive pion. Moreover, the pion and the quark masses are expected to obey the GOR relation from the chiral Lagrangian consideration. To derive the GOR relation from first principles, we should have a good control on the strong coupling dynamics of QCD. Relying on holographic principle, our analysis verifies the GOR relation impressively. Furthermore, we find a good numerical agreement of the chiral condensate with the experimental or lattice results.

In our construction, by introducing additional technicolor branes, a novel mechanism of worldsheet instanton gives us a controlled contribution to the quark mass deformation. In the strong coupling regime, the N_c color branes are replaced by a curved geometry, and the corresponding worldsheet instanton is now dressed by the Wilson line of the probe D8-brane gauge field via worldsheet boundary interactions. This results in an additional term in the D8-brane effective action corresponding to the mass deformation of the chiral Lagrangian of the pions.

We realize our idea with different types of D-branes, the D4′- and D6-branes. The former has a clear field theory interpretation in terms of a GUT-like extended technicolor theory, while the latter case is more tractable in actual calculations. Moreover, it allows us flavor-dependent quark mass terms. QCD θ angle can also be introduced easily by turning on $C^{(2)}_{\rm RR}$ in the background. Based on these brane settings, we have verified that the deformations indeed correspond to the lowest mass perturbation in the chiral Lagrangian of pions via a novel mechanism of worldsheet instantons. We also have the GOR relation satisfied, and from this relation we have extracted the quark masses and the chiral condensate of the Sakai-Sugimoto model for the first time, which happens to be surprisingly close to the lattice QCD estimate.

JHEP07(2008)089

We expect that the progress made in this paper can push the Sakai-Sugimoto model towards a more realistic nonperturbative description of QCD. We hope it may inspire more comparisons with the experimental or lattice QCD results. For example, we may expect the worldsheet instanton will also affect other hadron spectrum and their dynamics simply because both mesons and baryons are excitations of the quarks. As shown in [3], there is a difficulty in fitting both the baryon and meson spectrum well at the same time in the Sakai-Sugimoto model. It is then interesting to see if our mass deformation would help to resolve the issue.

There are many issues in lattice QCD or nuclear physics which call for a reliable and calculable QCD model with non-zero quark masses. For example, the mass shift of the pions in a finite temperature or in quark-gluon plasma phase is of much interest. These can be done only with the models with non-zero quark mass as the one we proposed. The study of renormalization of the value of chiral condensate is also interesting. Also, lattice computations have been done in quench approximation where quarks are heavy. It is often stated that the Sakai-Sugimoto model corresponds to a quenched approximation. However, it is different from the lattice quenching of heavy quarks, because in the original Sakai-Sugimoto model quarks are massless. It is also hoped that our mass deformation will help the holographic QCD to address some issues like nuclear potential in nuclear physics, where the pion mass will play an important role in their dynamics.

Note added during proofs. While we are preparing our manuscript, there appeared a paper [21] which has some overlap with ours in discussing the mass deformation by the worldsheet instantons.

Acknowledgments

T.H. would like to thank Kazuyuki Furuuchi, Seiji Terashima and Dan Tomino. H.U.Y. would like to thank Francesca Borzumati and High Energy Group in National Central University, Taiwan for an invitation. K.H. is grateful to members of String theory group in Taiwan for hospitality, and also thanks H. Suzuki and A. Miwa for valuable discussions. This work is partially supported by the Japan Ministry of Education, Culture, Sports, Science and Technology, Taiwan's National Center for Theoretical Sciences and National Science Council (No. NSC 97-2119-M-002-001, NSC 96-2112-M-003-014).

A. Field-theoretical computation of the four-Fermi term

In this appendix, we compute the four-Fermi coupling constant in a field theory perturbation, to illustrate the exponential factor in the worldsheet instanton amplitude (2.6).

The theory living on the D4-branes and the D4′-branes is 5 dimensional. The four-Fermi coupling appears effectively after one integrates out the massive W-bosons. The coupling between the W-boson and quarks (techni-quarks) is given by gauge couplings

$$\frac{g}{2}\int d\tau d^4x \left[\delta(\tau)\bar{q}_L\gamma_\mu W^\mu Q_L + \delta(\tau-\pi/M_{\rm KK})\bar{q}_R\gamma_\mu W^\mu Q_R\right]. \tag{A.1}$$

Note that the delta-functions are inserted in the coupling. The N_f D8-branes are located at $\tau = 0$ while the N_f $\overline{\text{D8}}$-branes are located at the anti-podal point, $\tau = \pi/M_{\text{KK}}$. On the D8-D4 intersection, the quark q_L lives, while on the D8-D4$'$ intersection, the quark Q_L lives. As for the intersection with the $\overline{\text{D8}}$-branes, q_R and Q_R live in the same manner. This zero-mode condition for the quarks is represented by the delta-functions above.

The coupling (A.1) is gauge invariant and also invariant under the chiral flavor symmetry $\text{U}(N_f)_L \times \text{U}(N_f)_R$. The coefficient g is the gauge coupling of the 5-dimensional Yang-Mills theory on the D4-branes and the D4$'$-branes, and so it is related to the 4-dimensional Yang-Mills coupling constant by a dimensional reduction, $(2\pi/M_{\text{KK}})(1/g^2) = 1/g_{\text{YM}}^2$, that is to say, a KK zero mode in the KK reduction along the direction τ is the gluon in 4-dimensions.

To derive the effective four-Fermi coupling, we decompose the 5-dimensional notation (A.1) into 4-dimensional fields, following the standard prescription given by [15]. We just employ a KK expansion of all the fields and compute the expanded theory as if it were a theory of infinite number of 4-dimensional fields. The KK expansion for the W-boson is given with the periodic boundary condition as[16]

$$W_\mu(x,\tau) = \sum_{n=0}^{\infty} \left[\sin(n\tau M_{\text{KK}}) W_\mu^{(n)}(x) + \cos(n\tau M_{\text{KK}}) \tilde{W}_\mu^{(n)}(x) \right]. \tag{A.2}$$

From this expression, it is obvious that the mass of the n-th KK W-boson field $W_\mu^{(n)}$ and $\tilde{W}_\mu^{(n)}$ is given by $(m^{(n)})^2 = M_W^2 + n^2 M_{\text{KK}}^2$. When we substitute the expansion (A.2) to the coupling (A.1), we obtain two couplings

$$\frac{g}{2} \sum_n \int d^4x\; \bar{q}_L \gamma^\mu \tilde{W}_\mu^{(n)} Q_L, \quad \frac{g}{2} \sum_n (-1)^n \int d^4x\; \bar{Q}_R \gamma^\mu \tilde{W}_\mu^{(n)} q_R. \tag{A.3}$$

In the latter coupling, the factor $(-1)^n$ comes from $\cos(\pi n)$ which indicates that this coupling resides on the point $\tau = \pi/M_{\text{KK}}$.

Next, from these couplings, we integrate out the massive W-bosons. By Wick-contracting the $\tilde{W}$ field in the couplings (A.3), we obtain a four-Fermi coupling,

$$-2 \int d^4x \int d^4y \int \frac{d^4p}{(2\pi)^4} \left[\frac{M_{\text{KK}}}{2\pi} \frac{1}{p^2 + M_W^2} + \frac{M_{\text{KK}}}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{p^2 + M_W^2 + n^2 M_{\text{KK}}^2} \right] \\ \times e^{ip\cdot(x-y)} \times g^2 \bar{q}_L(x) q_R(y) \bar{Q}_R(x) Q_L(y). \tag{A.4}$$

We have used a Fierz identity. Note that the first factor $M_{\text{KK}}/(2\pi)$ comes as a normalization of the kinetic term of the zero mode in the KK expansion, $\int d\tau\; 1^2 = 2\pi/M_{\text{KK}}$, while the factor M_{KK} in the second term comes similarly as $\int_0^{2\pi/M_{\text{KK}}} d\tau\; \cos^2(n\tau M_{\text{KK}}) = \pi/M_{\text{KK}}$. In (A.4), let us assume a large M_W, so the momentum can be neglected, $-\partial^2 \equiv p^2 \ll M_W^2$. Then we get the expression

$$-2 \left[\frac{M_{\text{KK}}}{2\pi} \frac{1}{M_W^2} + \frac{M_{\text{KK}}}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{M_W^2 + n^2 M_{\text{KK}}^2} \right] \int d^4x\; g^2 \bar{q}_L(x) q_R(x) \bar{Q}_R(x) Q_L(x). \tag{A.5}$$

[16] We neglect the component $\mu = \tau$ of the W-boson for simplicity. It is just equivalent to an adjoint scalar field, with which the computations presented below will follow in the same manner.

Using the formula

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+s^2} = -\frac{1}{2s^2} + \frac{\pi}{2s}\frac{1}{\sinh s\pi}, \tag{A.6}$$

The four-Fermi coefficient is evaluated as

$$-g^2 \frac{1}{M_W} \frac{1}{\sinh(\pi M_W/M_{\rm KK})} \int d^4x \; \bar{q}_L(x) q_R(x) \bar{Q}_R(x) Q_L(x). \tag{A.7}$$

When the techni-quarks Q on the D4′-branes get condensed to form $\langle \bar{Q}_L Q_R \rangle$, this four-Fermi term generates a mass term for the quarks. In our approximation, the effectiveness requires $M_W \gg M_{\rm KK}$. In this parameter region, the quark mass is obtained as[17]

$$m_q = \frac{g^2 \langle \bar{Q}_R Q_L \rangle}{2M_W} \exp\left[-\frac{\pi M_W}{M_{\rm KK}}\right] = \frac{\pi g_{\rm YM}^2 \langle \bar{Q}_R Q_L \rangle}{M_{\rm KK} M_W} \exp\left[-\frac{\pi M_W}{M_{\rm KK}}\right]. \tag{A.8}$$

The exponential factor is important, it coincides with our worldsheet instanton calculation (2.6).

However, note that the coefficient in front of the exponential factor in (A.8) is not necessarily coincident with the coefficient c in (2.6). The calculation of c is for a strongly coupled technicolor sector with the use of the AdS/CFT for the sector, while here we demonstrated only in the weak coupling region of the technicolor sector (and we secretly replaced $\bar{Q}_R Q_L$ with its condensate, by hand). This condensation is expected to occur only at the strong coupling region, so our calculation in this appendix is just for an illustration.

B. Curved shape of the probe D6-brane

In the flat spacetime background, the probe D6-brane is flat. But once we replace the N D4-branes by its geometry, the worldvolume of the D6-brane is curved due to the force between the D6-branes and the D4-branes, so that it is stable. Let us calculate this stable configuration. We introduce spherical coordinates for the directions transverse to the D4-brane,

$$x^5 = U\cos\theta_1, \quad x^6 = U\sin\theta_1\cos\theta_2, \ldots \tag{B.1}$$

To avoid difficulties in computation, we make an approximation that the D6-brane configuration is almost the same as that of the D6-brane not ending on the D8-brane but wrapping the x^4 circle. We choose the following ansatz for the D6-brane worldvolume parameterized by $(\sigma_1, \sigma_2, x^{0,1,2,3,4})$,

$$\theta_3 = \theta_4 = 0, \quad \theta_2 = \sigma_2, \quad \theta_1 = \sigma_1, \quad U = U(\sigma_1) \tag{B.2}$$

which is maximally rotation-symmetric. The D6-brane effective action, which is a 7-dimensional DBI action in the Witten's background spacetime, is given by

$$S = \mathcal{T}_{\rm D6} \int d\sigma_1 d\sigma_2 d^5x e^{-\phi}\sqrt{-\det g} = V \int d\theta_1 \; U^{5/2}\sin\theta_1 \sqrt{\left(\frac{dU}{d\theta_1}\right)^2 + U^2 f(U)}, \tag{B.3}$$

[17] In the evaluation we drop trivial overall numerical factors.

where $V = R^{-3/2}\mathcal{T}_{\rm D6}\int d^5x$. We can easily solve the equation of motion of this action numerically, with the initial condition

$$\mathrm{U}(\theta_1 = 0) = U_0, \quad \left.\frac{dU}{d\theta_1}\right|_{\theta_1=0} = 0. \tag{B.4}$$

Here U_0 is a constant parameter. This U_0 is the minimum distance between the D6-brane and the D4-branes. Our numerical result shows a consistent configuration of a curved D6-brane in the background. The worldvolume point $\theta_1 = 0$ is the closest to the D4-branes. It is the same as the flat configuration of the D6-brane in the flat background.

C. Two-instantons and π^0-$\pi^\pm$ mass difference

The two instanton sector is described in the following way. The instanton number is just the wrapping number of the boundary of the worldsheet on the non-trivial cycle on the D8-brane. When it winds once, the instanton contribution is proportional to (3.10). So, the two instanton sector is given by

$$\sum_i\sum_j \mathrm{tr}\left[\mathrm{P}\exp\left[-\Phi\int_b dz\right]\cdot\mathrm{P}\exp\left[-i\oint_i A_z dz\right]\cdot\mathrm{P}\exp\left[-\Phi\int_b dz\right]\cdot\mathrm{P}\exp\left[-i\oint_j A_z dz\right]\right] \tag{C.1}$$

Here, again we have followed the approximation that the integral regions of Φ is separated from the integral region of A_z. Then, using (3.11) and looking at the front factor in the one-instanton result (3.12), we obtain

$$S_{2-\mathrm{instanton}} = \frac{1}{3^{9/2}\pi^3} g_{\rm YM}^3 N_c^{3/2} M_{\rm KK}^3 \mathcal{N}^{-2}\cdot\frac{g_{\rm YM}^2}{2\pi M_{\rm KK}}\int d^4x \mathrm{tr}\left[m_q U m_q U + U^\dagger m_q U^\dagger m_q\right] \tag{C.2}$$

This expression is, again, consistent with chiral perturbation theory. We expand this expression with $U = \exp[2i\pi(x)/f_\pi]$, then we obtain terms quadratic in the pion fields as

$$\begin{aligned}\mathrm{tr}\left[m_q U m_q U + U^\dagger m_q U^\dagger m_q\right]_{\mathcal{O}(\pi^2)} &= -\frac{8}{f_\pi^2}\mathrm{tr}\left[m_q\pi(x)m_q\pi(x) + m_q^2\pi(x)^2\right] \\ &= -\frac{8}{f_\pi^2}\left((m_u^2+m_d^2)(\pi^0(x))^2+\frac{1}{2}(m_u+m_d)^2((\pi^+(x))^2+(\pi^-(x))^2)\right)\end{aligned}$$

where $\pi^\pm \equiv (\pi_1 \pm i\pi_2)/\sqrt{2}$ and $\pi^0 = \pi_3$. From this, the mass difference is obtained as

$$|m_{\pi^\pm}^2 - m_{\pi^0}^2| = \frac{2}{3\sqrt{3}}\frac{g_{\rm YM}^3}{\sqrt{N_c}\mathcal{N}^2}(m_u - m_d)^2. \tag{C.3}$$

Let us give a numerical estimate as an illustration. If we substitute $N_c = 3$, $g_{\rm YM} = 2.35$ and also the experiment values of the pion masses, $m_{\pi^\pm} = 139.6$ [MeV], $m_{\pi^0} = 135.0$ [MeV], then with $\mathcal{N} = 1$, we obtain $|m_u - m_d| \simeq 21$ [MeV]. This is larger than the experimental values of the quark masses.

The origin of this discrepancy can be understood as follows. In a chiral perturbation theory, there are other higher order terms, $(\mathrm{tr}[m_q U])^2$ and $(\mathrm{tr}[m_q U^\dagger])^2$. Together with these terms, the realistic pion mass difference is reproduced. In our holographic QCD approach, these double-trace operators appear at a sub-leading order in the large N expansion and so does not appear in our leading-order estimates. It would be interesting to compute these sub-leading corrections and see how the masses of $K^{0,\pm}$ can be reproduced from strange quark mass.

References

[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [`hep-th/9711200`].

[2] T. Sakai and S. Sugimoto, *Low energy hadron physics in holographic QCD*, *Prog. Theor. Phys.* **113** (2005) 843 [`hep-th/0412141`]; *More on a holographic dual of QCD*, *Prog. Theor. Phys.* **114** (2006) 1083 [`hep-th/0507073`].

[3] D.K. Hong, M. Rho, H.-U. Yee and P. Yi, *Chiral dynamics of baryons from string theory*, *Phys. Rev.* **D 76** (2007) 061901 [`hep-th/0701276`]; *Dynamics of baryons from string theory and vector dominance*, *JHEP* **09** (2007) 063 [`arXiv:0705.2632`];
H. Hata, T. Sakai, S. Sugimoto and S. Yamato, *Baryons from instantons in holographic QCD*, `hep-th/0701280`;
K. Nawa, H. Suganuma and T. Kojo, *Baryons in holographic QCD*, *Phys. Rev.* **D 75** (2007) 086003 [`hep-th/0612187`].

[4] K. Hashimoto, C.-I. Tan and S. Terashima, *Glueball decay in holographic QCD*, *Phys. Rev.* **D 77** (2008) 086001 [`arXiv:0709.2208`].

[5] M. Gell-Mann, R.J. Oakes and B. Renner, *Behavior of current divergences under* $SU(3) \times SU(3)$, *Phys. Rev.* **175** (1968) 2195.

[6] K. Hashimoto, T. Hirayama and A. Miwa, *Holographic QCD and pion mass*, *JHEP* **06** (2007) 020 [`hep-th/0703024`].

[7] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [`hep-th/9802109`];
E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [`hep-th/9802150`].

[8] R. Casero, E. Kiritsis and A. Paredes, *Chiral symmetry breaking as open string tachyon condensation*, *Nucl. Phys.* **B 787** (2007) 98 [`hep-th/0702155`];
O. Bergman, S. Seki and J. Sonnenschein, *Quark mass and condensate in HQCD*, *JHEP* **12** (2007) 037 [`arXiv:0708.2839`];
A. Dhar and P. Nag, *Sakai-Sugimoto model, tachyon condensation and chiral symmetry breaking*, *JHEP* **01** (2008) 055 [`arXiv:0708.3233`].

[9] S. Weinberg, *Implications of dynamical symmetry breaking*, *Phys. Rev.* **D 13** (1976) 974; *Implications of dynamical symmetry breaking: an addendum*, *Phys. Rev.* **D 19** (1979) 1277;
L. Susskind, *Dynamics of spontaneous symmetry breaking in the Weinberg-Salam theory*, *Phys. Rev.* **D 20** (1979) 2619.

[10] S. Dimopoulos and L. Susskind, *Mass without scalars*, *Nucl. Phys.* **B 155** (1979) 237;
E. Eichten and K.D. Lane, *Dynamical breaking of weak interaction symmetries*, *Phys. Lett.* **B 90** (1980) 125.

[11] T. Hirayama and K. Yoshioka, *Holographic construction of technicolor theory*, *JHEP* **10** (2007) 002 [`arXiv:0705.3533`].

[12] E. Witten, *Anti-de Sitter space, thermal phase transition and confinement in gauge theories*, *Adv. Theor. Math. Phys.* **2** (1998) 505 [hep-th/9803131].

[13] G. 't Hooft, *A planar diagram theory for strong interactions*, *Nucl. Phys.* **B 72** (1974) 461.

[14] M. Kruczenski, D. Mateos, R.C. Myers and D.J. Winters, *Towards a holographic dual of large-N_c QCD*, *JHEP* **05** (2004) 041 [hep-th/0311270].

[15] E.A. Mirabelli and M.E. Peskin, *Transmission of supersymmetry breaking from a 4-dimensional boundary*, *Phys. Rev.* **D 58** (1998) 065002 [hep-th/9712214].

[16] G. Aldazabal, S. Franco, L.E. Ibáñez, R. Rabadán and A.M. Uranga, *Intersecting brane worlds*, *JHEP* **02** (2001) 047 [hep-ph/0011132].

[17] C.G. Callan and J.M. Maldacena, *Brane dynamics from the Born-Infeld action*, *Nucl. Phys.* **B 513** (1998) 198 [hep-th/9708147].

[18] O. Bergman and G. Lifschytz, *Holographic* $\mathrm{U}(1)_A$ *and string creation*, *JHEP* **04** (2007) 043 [hep-th/0612289].

[19] O. Aharony, J. Sonnenschein and S. Yankielowicz, *A holographic model of deconfinement and chiral symmetry restoration*, *Ann. Phys. (NY)* **322** (2007) 1420 [hep-th/0604161].

[20] S.R. Coleman and E. Witten, *Chiral symmetry breakdown in large N chromodynamics*, *Phys. Rev. Lett.* **45** (1980) 100.

[21] O. Aharony and D. Kutasov, *Holographic duals of long open strings*, arXiv:0803.3547.